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EXAMINER

LANGEL, W

ART. UNIT	PAPER NUMBER
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1754

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Please find below and/or attached an Office communication concerning this application or proceeding.

Commissioner of Patents and Trademarks

# Office Action Summary

Application No.

009837

Applicant(s)

Mills

Examiner

Langel

Group Art Unit

1754

—The MAILING DATE of this communication appears on the cover sheet beneath the correspondence address—

## Period for Response

A SHORTENED STATUTORY PERIOD FOR RESPONSE IS SET TO EXPIRE 3 MONTH(S) FROM THE MAILING DATE OF THIS COMMUNICATION.

- Extensions of time may be available under the provisions of 37 CFR 1.136(a). In no event, however, may a response be timely filed after SIX (6) MONTHS from the mailing date of this communication.
- If the period for response specified above is less than thirty (30) days, a response within the statutory minimum of thirty (30) days will be considered timely.
- If NO period for response is specified above, such period shall, by default, expire SIX (6) MONTHS from the mailing date of this communication.
- Failure to respond within the set or extended period for response will, by statute, cause the application to become ABANDONED (35 U.S.C. § 133).

## Status

- ☐ Responsive to communication(s) filed on \_\_\_\_\_.
- ☐ This action is FINAL.
- ☐ Since this application is in condition for allowance except for formal matters, prosecution as to the merits is closed in accordance with the practice under *Ex parte Quayle*, 1935 C.D. 11; 453 O.G. 213.

## Disposition of Claims

- ☒ Claim(s) 17-299 is/are pending in the application.
- ☐ Of the above claim(s) \_\_\_\_\_ is/are withdrawn from consideration.
- ☐ Claim(s) \_\_\_\_\_ is/are allowed.
- ☒ Claim(s) 17-299 is/are rejected.
- ☐ Claim(s) \_\_\_\_\_ is/are objected to.
- ☐ Claim(s) \_\_\_\_\_ are subject to restriction or election requirement.

## Application Papers

- ☐ See the attached Notice of Draftsperson's Patent Drawing Review, PTO-948.
- ☐ The proposed drawing correction, filed on \_\_\_\_\_ is ☐ approved ☐ disapproved.
- ☐ The drawing(s) filed on \_\_\_\_\_ is/are objected to by the Examiner.
- ☐ The specification is objected to by the Examiner.
- ☐ The oath or declaration is objected to by the Examiner.

## Priority under 35 U.S.C. § 119 (a)-(d)

- ☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d).
  - ☐ All ☐ Some\* ☐ None of the CERTIFIED copies of the priority documents have been received.
  - ☐ received in Application No. (Series Code/Serial Number) \_\_\_\_\_.
  - ☐ received in this national stage application from the International Bureau (PCT Rule 1.7.2(a)).

\*Certified copies not received: \_\_\_\_\_

## Attachment(s)

- ☐ Information Disclosure Statement(s), PTO-1449, Paper No(s). \_\_\_\_\_
- ☒ Notice of References Cited, PTO-892
- ☐ Notice of Draftsperson's Patent Drawing Review, PTO-948
- ☐ Interview Summary, PTO-413
- ☐ Notice of Informal Patent Application, PTO-152
- ☐ Other \_\_\_\_\_

Office Action Summary

Art Unit 1754

35 U.S.C. § 101 REJECTION

35 U.S.C. § 101 reads as follows:

Whoever invents or discovers any new and useful process, machine, manufacture, or composition of matter, or any new and useful improvement thereof, may obtain a patent therefor, subject to the conditions and requirements of this title.

Claims 17-299 are rejected under 35 U.S.C. § 101 because the disclosed invention is inoperative and therefore lacks credible utility. All the claims recite "hydrino hydride ion." Page 4 of applicant's specification defines "hydrino atom" or "hydrino" as a "hydrogen atom having the binding energy given in Eq.(2)", wherein Eq.(2) recites

$$\text{Binding Energy} = \frac{13.6 \text{ eV}}{n^2}$$

where  $n = 1/p$  and  $p$  is an integer greater than one. A "hydrino hydride" would thus constitute a hydrogen atom having new energy states that are below the conventionally accepted ground state energy. An asserted utility would not be considered credible where a person of ordinary skill would consider the assertion to be incredible in view of contemporary knowledge and where the evidence offered by applicant does not counter what contemporary knowledge otherwise suggests. See MPEP § 2107.01. See the attached Appendix which shows the mathematical justification as

Art Unit 1754

to why conventional theory and experiment preclude the existence of hydrino atoms. It is emphasized that Endnote 1 of the Appendix shows that Schrodinger's wave equation mandates that the value of "n" must be a positive integer having the values 1, 2, 3, and so on, and Endnote 5 shows that fractional values for "n" are also impermissible in light of the Uncertainty Principle. The fourth full paragraph on page 19-14 of Bethe & Salpeter's Quantum Mechanics of One-and Two-Electron Atoms (Plenum Publishing Corporation, New York, 1977) states that the "ground state" of hydrogen has  $n = 1$ . It is clear from the foregoing that fractional values for "n" cannot exist according to conventional scientific theories. Once the Patent and Trademark Office shows through scientific reasoning that an invention is inoperative, the burden then shifts to applicant to provide satisfactory evidence of operability of the invention. Newman v. Quigg, 877 F. 2d 1575, 11 USPQ 2d 1340 (Fed. Cir. 1989).

35 U.S.C. § 112 REJECTION

Claims 17-299 are rejected under 35 U.S.C. § 112, first paragraph, as containing subject matter which was not described in the specification in such a way as to enable one skilled in the art to which it pertains, or with which it is most nearly connected, to make and/or use the invention. The specification does not enable one of ordinary skill in the art to make or use a "hydrino hydride ion", in that it would require undue

Art Unit 1754

experimentation to do so. Factors to be considered in determining whether a disclosure would require undue experimentation include (1) the quantity of experimentation necessary, (2) the amount of direction or guidance presented, (3) the presence or absence of working examples, (4) the nature of the invention, (5) the state of the prior art, (6) the relative skill of those in the art, (7) the predictability or unpredictability of the art, and (8) the breadth of the claims. In re Wands, 858 F. 2d 731, 737, 8 USPQ 2d 1400, 1404 (Fed. Cir. 1988). Each of these factors outlined in Wands will be addressed as to their relevance to the lack of enablement for applicant's claims.

(1) The Quantity of Experimentation Necessary

Pages 65-68 of applicant's specification show that hydrino hydride was prepared during the electrolysis of an aqueous solution of  $K_2CO_3$  corresponding to the catalyst  $K^+/K^+$ . However U.S. Patent 4,337,126 (Gilligan, III et al. ) (newly cited) is evidence that the electrolysis of potassium carbonate results in the production of potassium hydroxide and  $CO_2$ . (See especially column 6, lines 13-47 of Gilligan, III et al.) Lines 6-9 on page 68 of the specification state that "the sample was prepared by filtering the  $K_2CO_3$  electrolyte from the cell described herein that produced  $6.3 \times 10^8$  J of enthalpy of formation of hydrino

Art Unit 1754

hydride with a Whatman 110 mm filter paper (Cat. No. 1450 110) to obtain white crystals", and lines 13-17 state that "the sample was prepared by acidifying the  $K_2CO_3$  electrolyte from the cell described herein that produced  $6.3 \times 10^8$  J of enthalpy of formation of hydrino hydride with  $HNO_3$ , and concentrating the acidified solution until yellow-white crystals formed on standing at room temperature." However, there are not sufficient details of the electrolysis conditions set forth on pages 65-68 which would allow one to simply filter the  $K_2CO_3$  electrolyte or acidify the  $K_2CO_3$  electrolyte with  $HNO_3$  to obtain the hydrino hydride sample, rather than simply produce potassium hydroxide and  $CO_2$  by the potassium carbonate electrolysis, as shown by Gilligan, III et al. In this regard, the specification must teach one of ordinary skill in the art how to make and use the invention, and not simply how to direct one how to find out how to make and use for himself. In re Gardner, 427 F. 2d 786, 789, 166 USPQ 138, 141 (CCPA 1970).

(2) The Amount of Direction or Guidance Presented

The direction or guidance provided in the specification is found on pages 65-68, which is insufficient for the same reasons given hereinbefore with respect to factor (1). It would require undue experimentation for one of ordinary skill in the art to make and use the claimed invention. In short, the amount of

Art Unit 1754

direction or guidance is insufficient to obtain the claimed invention, as Gilligan, III et al. shows that electrolysis of potassium carbonate would be expected to simply result in the production of potassium hydroxide and  $\text{CO}_2$ .

(3) The Presence or Absence of Working Examples

The specification contains, on pages 64-109, examples of methods for forming and identifying hydrino hydride ions, which are a type of "increased binding energy" hydrogen species as recited in applicant's claims. It is unclear however whether applicant has actually formed and identified the variously recited species, since the Examples are directed to the electrolysis of aqueous  $\text{K}_2\text{CO}_3$ , which would, as stated above, produce KOH and  $\text{CO}_2$ . The present examples are thus not considered to be working examples.

(4) The Nature of the Invention

The scientific community has held the belief for decades that hydrogen cannot exist below the "ground state" ( $n = 1$ ). (See the reasoning presented hereinbefore with respect to the rejection under 35 U.S.C. § 101 for inoperability.) Accordingly, the nature of the invention is such that it would be startling if it were operative, thus requiring greater detail than that found on pages 65-108 of the specification for one of ordinary skill in

Art Unit 1754

the art to make and use the claimed invention without undue experimentation. Applicant himself points out that the Mills theory predicts the existence of a previously unknown form of matter: hydrogen atoms and molecules having electrons of lower energy than the conventional "ground" state, called "hydrinos" and "dihydrinos", respectively, where each energy level corresponds to a fractional quantum number. (See the paragraph bridging pages 13 and 14 of R. L. Mills, The Grand Unified Theory of Classical Quantum Mechanics (Black Light Power, Inc., New Jersey, 1999)).

(5) The State of the Prior Art

There appears to be no prior art showing hydrogen with a quantum number below 1, or even any prior art which would suggest that hydrogen with a quantum number below 1 could even exist in theory. The closest prior art to that disclosed in applicant's specification (see Gilligan, III et al., for example) show that hydrino hydride ions would not be formed. Also, see the attached Appendix. Applicant himself points out that the Mills theory predicts the existence of a previously unknown form of matter: hydrogen atoms and molecules having electrons of lower energy than the conventional "ground" state, called "hydrinos" and "dihydrinos", respectively, where each energy level corresponds to a fractional quantum number. (See the paragraph bridging



Art Unit 1754

pages 13 and 14 of R. L. Mills, The Grand Unified Theory of Classical Quantum Mechanics (Black Light Power, Inc., New Jersey, 1999)).

(6) The Relative Skill of Those in the Art

Even the most highly skilled physicists were of the opinion that hydrogen cannot exist below the "ground state" ( $n = 1$ ). See the attached Appendix.

(7) The Predictability or Unpredictability of the Art

It would be most unpredictable that the hydrogen atom could exist below the "ground state" ( $n = 1$ ). (See the reasoning presented hereinbefore with respect to the rejection under 35 U.S.C. § 101 for inoperability and the attached Appendix.)

(8) The Breadth of the Claims

Since all the claims require the presence of at least one hydrino hydride ion, and it has been shown hereinbefore with respect to the rejection under 35 U.S.C. § 101 for inoperability that the hydrino hydride ion cannot exist, the claims are not enabled.

Considering all of the above factors, one skilled in the art could not make and/or use the claimed invention without undue experimentation.

Serial No. 09/009,837

-8-

Art Unit 1754

Any inquiry concerning this communication should be directed to Wayne A. Langel at telephone number (703) 308-0248.

WAL:cdc

September 7, 2000

*Wayne A. Langel*  
WAYNE LANGEL  
PRIMARY EXAMINER  
GROUP 110

## Appendix

Applicant has referred to the book by R. L. Mills entitled *The Grand Unified Theory of Classical Quantum Mechanics* (Blacklight Power Inc., New Jersey, 1999; hereafter, "GUT") which describes the existence of new energy states for the hydrogen atom that are below the conventionally accepted ground state energy. A hydrogen atom in any one of these new energy states is termed a "hydrino." According to equations (I.75a-c) on pages 19-20 of GUT, the general formula representing the energy levels for an electron with a principal quantum number,  $n$ , around the nucleus of the hydrogen atom is:

$$E_n = - (\text{Rydberg constant})/n^2 = - 13.6 \text{ electron volt}/n^2$$

where  $n = 1, 2, 3$ , etc and  $n$  is also  $= 1/2, 1/3, 1/4$ , etc. While the former integer values of  $n$  give energies that are conventionally understood and experimentally verified, the latter fractional values of  $n$  lead to the energies of the electron in a hydrino atom which, according to Mills, represents a new "lower energy hydrogen atom."

A review of some of the main mathematical underpinnings in GUT shows that there is really no proper theoretical basis to assert the existence of the hydrino atom in view of the following discussion.

Nowhere has Mills satisfactorily established that fractional values of  $n$  arise as a natural consequence of a logical and internally consistent mathematical and scientific framework. While GUT bristles with a dense array of mathematical equations, the fractional values of  $n$  are not shown to be the unequivocal end result of Mills' theory. It appears that there is an internal break in logic in the mathematical analysis, with Mills ultimately relying on conclusionary statements, such as, a nonradiative boundary condition and the relationship between the electron and a photon gives transitions in which the electron goes to a "lower" energy nonradiative state with a smaller radius or, alternatively, that an electron can undergo a collision with an "energy hole" which allows the electron to undergo a transition to a lower energy nonradiative state with a smaller radius (pages 16-17 of GUT). In these transitions,

the process involved is called a “shrinkage reaction” yielding a shrunken hydrogen atom accompanied by the release of energy. See pages 16, 17 and 144-146 of GUT.

By way of background, it is noted that there are at least two conventionally recognized approaches to the problem of obtaining the energy levels of the electron in the hydrogen atom. These are:

(a) *Via* a Differential Equation approach formulated as a two-point boundary value problem where boundary conditions at the nucleus and at infinity are imposed on the radial wavefunction of the electron which satisfies a second-order linear differential equation known as Schrödinger’s wave equation. It is to be understood that while the complete wavefunction in spherical polar coordinates is the product of a radial wavefunction and angular wavefunctions, the complete wavefunction for the ground (or lowest energy) state of the hydrogen atom is independent of angular coordinates in view of the spherical symmetry of that state, and is studied only on the basis of the radial wavefunction. Thus, see attached sections 18d-18e and 21b at pages 121-124 and 139 from Pauling and Wilson’s *Introduction to Quantum Mechanics* (Dover Publications, Inc., New York, 1985) and **Endnote 1**.

(b) *Via* an Integral Equation approach wherein the boundary conditions on the radial wavefunction of the electron are “built into” the integral equation itself rather than being imposed on it as in the differential equation formulation. In this approach, upon taking the Fourier transform of the wavefunction, subject to the boundary condition that it satisfies Schrödinger’s equation, an integral equation is obtained. Thus, see attached pages 899-900 from Morse and Feshbach’s *Methods of Theoretical Physics, Part I* (McGraw-Hill Book Company, New York, 1953) and **Endnote 2**.

It is crucial to note that either approach is but a mathematical tool and that, while the integral equation approach may be mathematically more compact and, perhaps, be more convenient for solving certain problems compared to the differential equation approach, the

final results given by either approach must not be mutually contradictory if a scientific theory based on these approaches is to be logical and internally consistent.

From a consideration of Mills' mathematical derivations on pages 4-5 (equations (I.5) to (I.11)) , on pages 32-38 (equations (1.3) to (1.45)) and on pages 136-141 (equations (5.1) to (5.21)) of GUT, it appears that Mills' formulation *may* be an integral equation type of approach. Specifically, the boundary condition "built into" the integral equation is an expression for the current density, and, thus, the charge density of a point charge which satisfies Maxwell's equation for the electric field as given by Haus in a paper, in the *American Journal of Physics*, vol. 54, no. 12, pages 1126-1129 (1986), relating to the absence of radiation from a point charge moving at constant velocity. See page 3 of GUT. While, Haus' paper is not the focus of discussion here, it is apparent that the use of a Dirac delta function,  $\delta(\mathbf{r}-\mathbf{r}_n)$ , to represent the electron charge density on page 4 of GUT may be an unphysical assumption in that, whereas the electron charge density is an "observable" that is ultimately measurable, the delta function, which purports to represent it, is not, in and of itself, a function in the usual mathematical sense of the term and is physically meaningful only under an integral sign.

More specifically, it appears that Mills' integral equation approach utilizes the technique of the "Green's function." In the theory of integral equations, the Green's function is a function that satisfies a differential equation involving a Dirac delta function type of point source. A connection between the Green's function and the wavefunction is established by requiring the former to satisfy boundary conditions corresponding to those satisfied by the latter. Interpreting Mills' equations as best as one can, it is possible, though by no means certain, that Mills achieves such a connection by requiring the Green's function to satisfy boundary conditions imposed on the charge density function in Mills' equation (1.1) on page 31 of GUT. The final step in the integral equation approach is to generate an integral

equation involving an integral taken over the Green's function. The solution of that equation would yield the wavefunction of the electron and, from that, leads to the energy levels of the electron in the hydrogen atom. See attached pages 808, 902 and 903 from Morse and Feshbach *op. cit.* and **Endnote 3**. It is observed that the legitimate use of a Green's function which satisfies an equation involving a Dirac delta function type of "point source" and appears, ultimately under an integral sign as the kernel of an integral equation, does not justify Mills' representation of the electron charge density, which is a "smeared out" charge distribution, as a Dirac delta function as discussed previously. Mills' lack of consistency in using properly subscripted variables as well as the absence of a logical flow in the mathematical derivations, prevents one from properly assessing the kind of approach taken in GUT.

In any event, at least some problematical issues are seen in the Mills' treatment, *viz.*, (i) it is not explained as to why it is physically meaningful to utilize Haus' boundary condition for a classical point charge moving in free space in order to obtain the energy levels of the electron in a quantized system such as the hydrogen atom where the electron moves in a confined space due to its attractive coulombic interaction with the positively charged nucleus, and, (ii) there is no explanation for the catastrophic collapse of the electron into the nucleus as  $n \rightarrow \infty$  in the fractional quantum number series,  $1/n$ , i.e. the hydrino atom implodes and ceases to exist. See pages 144-146 of GUT. The end result of Mills' integral equation approach, if such it is, fails to bear out his assertion that  $n$  must unequivocally have fractional values. In essence, it appears that the condition that  $n$  have fractional values (see equations (1.75c) and (2.2) on pages 20 and 81 of GUT) is but an *ad hoc* statement that does not logically flow from Mills' derivation of the equation for the energy levels of the electron in the hydrogen atom and it may even represent a type of forced parameterization scheme

- deliberately structured to produce a desired outcome contrary to the logical flow of its mathematics or, even, common sense.

Hence, it appears that Mills' theory remains essentially unproven as discussed above and does not constitute a proper basis to demonstrate the existence, at least on theoretical grounds alone, of the so-called hydrino atom.

Furthermore, Mills' theory does not show that the conventional quantum mechanical treatment of the hydrogen atom is theoretically or experimentally flawed. Any attempt to establish a new result for the hydrogen atom that is presently unknown to quantum mechanics must cross a rather steep threshold of scientific credibility. See the attached page 2 from Bethe and Salpeter's *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977 and **Endnote 4**.

Among the many problems solved by quantum mechanics, the hydrogen atom, along with the linear harmonic oscillator and the particle-in-a-box, is one of the few scientific problems that has received extensive theoretical and experimental treatment over many years since the first decade of the twentieth century. For a complete treatment of the hydrogen atom problem see the attachment from pages 19-1 to 19-18 of Feynman's *Lectures in Physics*, vol. III, *Quantum Mechanics* (Addison-Wesley Publishing Co., Reading, Mass., 1965). The results obtained from at least one type of standard procedure for solving the radial Schrödinger equation using a power series expansion for the wavefunction of the electron inescapably lead to the conclusion that only positive integer values for  $n$  are permissible (as explained previously in **Endnote 1**). See attached pages 1-9 and 2-6 from Feynman *op. cit.* and **Endnote 5**. In other words, conventional theory and experiment forbid hydrino atoms.

### Endnote 1

Schrödinger's wave equation for the radial wavefunction,  $S(\rho)$ , is:

$$(1/\rho^2)(d/d\rho)(\rho^2 dS/d\rho) + \{-1/4 - l(l+1)/\rho^2 + \lambda/\rho\} S = 0$$

where  $\rho$  is proportional to the radial coordinate in the spherical polar coordinate system with  $0 \leq \rho \leq \infty$ ,  $l$  is the orbital angular momentum quantum number and  $\lambda$  is proportional to negative (i.e. bound) energy values. The boundary conditions are that far from the nucleus of the hydrogen atom ( $\rho \rightarrow \infty$ ) the radial wavefunction becomes negligible i.e.  $S \rightarrow 0$ , and, at the nucleus of the atom ( $\rho = 0$ ), noting that  $S$  is expressible as  $e^{-\rho/2} \rho^s L(\rho)$  where  $L = \sum a_v \rho^v$  is an infinite power series in  $\rho$ , substitution of the expression for  $S$  into the radial wavefunction equation results in the choice of  $s = +l$  (which is a positive integer) as the only choice that will permit to be  $S$  be an acceptable wavefunction, which in turn yields the boundary condition that  $S$  has a finite value at the nucleus. Note that despite the finite value of the radial wavefunction at the nucleus, the probability of finding the electron at the nucleus,  $\rho = 0$ , of the hydrogen atom in its normal ground state is proportional to  $4\pi\rho^2 S^2$  which, of course, is zero. Upon substituting the cited expression for  $S$  into the radial wavefunction equation, recursion relations between  $a_v$  for various values of  $v$  are obtained. The recursion relations contain the principal quantum number  $n$  appearing as a multiplicative coefficient of  $a_v$ . Since  $S$  must have a proper asymptotic behavior as  $\rho \rightarrow \infty$ , this requires that the infinite power series be terminated after a finite number of terms which in turn, after some algebra, leads to the result that  $n$  must be a positive integer having the values 1, 2, 3, etc.. See equations (18.29) to (18.39) and Figure 21-1 at pages 121-124 and 140 in Pauling and Wilson.



### Endnote 2

Substitution of the Fourier transform of the wavefunction,  $\psi(\mathbf{r})$ , *viz.*

$$\psi(\mathbf{r}) = (1/h)^{3/2} \int_{-\infty}^{\infty} \varphi(\mathbf{p}) e^{(2\pi i/h)\mathbf{p}\cdot\mathbf{r}} d\mathbf{p},$$

where  $h$  is Planck's constant and  $\mathbf{p}$  and  $\mathbf{r}$  are momentum and spatial coordinate vectors, respectively, into the Schrödinger equation in the differential form

$$\nabla^2 \psi + (2m/h^2) \{E - V[\mathbf{r}, (h/2\pi i)\nabla]\} \psi = 0,$$

where  $\nabla^2$ ,  $E$  and  $V$  are the Laplacian operator, total and potential energies, respectively, followed by multiplication through by  $(1/h)^{3/2} e^{(-2\pi i/h)\mathbf{q}\cdot\mathbf{r}}$  and an integration over  $\mathbf{r}$  yields the desired integral equation

$$(q^2/2m)\varphi(\mathbf{q}) + \int_{-\infty}^{\infty} \varphi(\mathbf{p}) V(\mathbf{p}-\mathbf{q}, \mathbf{p}) d\mathbf{p} = E\varphi(\mathbf{q})$$

where

$$V(\mathbf{p}-\mathbf{q}, \mathbf{p}) = (1/h)^{3/2} \int_{-\infty}^{\infty} e^{(2\pi i/h)(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}} V(\mathbf{r}, \mathbf{p}) d\mathbf{r}$$

with  $\mathbf{q}$  being a momentum vector.

See equation (8.1.4) at page 900 in Morse and Feshbach.

### Endnote 3

To illustrate a method of obtaining a solution for the wavefunction,  $\psi$ , by the technique of Green's functions consider the Schrödinger equation for  $\psi$  written as:

$$[\nabla^2 + k^2]\psi = U\psi$$

where  $k^2 = (8\pi^2 m/h^2)E$  and  $U = (8\pi^2 m/h^2)V$  with  $E$  and  $U$  being the total and potential energies, respectively. A Green's function,  $G_k(\mathbf{r}|\mathbf{r}_0)$ , is introduced which satisfies

$$[\nabla^2 + k^2]G_k(\mathbf{r}|\mathbf{r}_0) = -4\pi\delta(\mathbf{r}-\mathbf{r}_0),$$

where  $\delta(\mathbf{r}-\mathbf{r}_0)$  is a Dirac delta function representing a "point source" at  $\mathbf{r}_0$ . The Green's function can be thought of as representing an effect at  $\mathbf{r}$  caused by a point source at  $\mathbf{r}_0$ . The

- boundary conditions on  $G_k(\mathbf{r}|\mathbf{r}_0)$  are chosen to be the same as those corresponding to the boundary conditions on the wavefunction  $\psi$ . Then, by the theory of integral equations, a solution to the Schrödinger equation is:

$$\psi(\mathbf{r}) = -(1/4\pi) \int G_k(\mathbf{r}|\mathbf{r}_0) U(\mathbf{r}_0) \psi(\mathbf{r}_0) d\mathbf{r}_0.$$

See pages 808, 902 and 903 in Morse and Feshbach.

#### Endnote 4

Regarding the study of the hydrogen atom, note the following quotation from page 2 of Bethe and Salpeter's classic text entitled *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977):

“One of the simplest, and most completely treated, fields of application of quantum mechanics is the theory of atoms with one or two electrons. For hydrogen and the analogous ions  $\text{He}^+$ ,  $\text{Li}^{++}$ , etc., the calculations can be performed exactly, both in Schrödinger's nonrelativistic wave mechanics and in Dirac's relativistic theory of the electron. More specifically, the calculations are exact for a single electron in a fixed Coulomb potential. Hydrogen-like atoms thus furnish an excellent way of testing the validity of quantum mechanics. For such atoms the correction terms due to the motion and structure of atomic nuclei and due to quantum electrodynamic effects are small and can be calculated with high accuracy. Since the energy levels of hydrogen and similar atoms can be investigated experimentally to an astounding degree of accuracy, some accurate tests of the validity of quantum electrodynamics are also possible.”

### Endnote 5

It is noteworthy that this position is also supported by a different line of argument that is independent of the solution to Schrödinger's equation. Thus, fractional values for the principal quantum number  $n$  would bring the electron much closer to the nucleus of the hydrogen atom than is permitted by Heisenberg's Uncertainty Principle. Feynman has presented a mathematically simple argument, in his "Lectures in Physics," vol. III, page 2-6, to show that the size of the hydrogen atom i.e. when  $n$  is 1 (rather than, say,  $1/2$ ) is perfectly consistent with the Uncertainty Principle. This argument goes as follows: from the Uncertainty Principle, if the electron is at a distance  $a$  from the hydrogen nucleus, then the product of its momentum and  $a$  must be of the order of Planck's constant. Now the total energy of the electron is the sum of its kinetic and potential energies. Noting that the kinetic energy can be expressed in terms of the square of the momentum, upon invoking the value of the momentum from the Uncertainty Principle and minimizing the total energy in order to obtain the lowest energy level of the electron, one immediately obtains the standard result for the lowest energy level of the electron in the hydrogen atom which is consistent with  $n$  being 1 and no lower than 1. Since, according to Feynman, "no one has ever found (or even thought of) a way around the Uncertainty Principle ... so we must assume it describes a basic characteristic of nature," (page 1-9 in Feynman) it appears that Mills' fractional value for  $n$  is impermissible in light of the inviolability of the Uncertainty Principle.

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